UPME

UNIDAD DE PLANEACIÓN MINERO-ENERGÉTICA

ESTUDIO SOBRE EL *"DESARROLLO DEL POTENCIAL DE COGENERACIÓN EN EL PAÍS"*

PLANTEAMIENTO MATEMATICO DEL MODELO DE DIMENSIONAMIENTO OPTIMO

AENE LTDA

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ÍNDICE DE MODIFICACIONES

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TABLA DE CONTENIDO

1. PLANTEAMIENTO MATEMÁTICO DEL MODELO DE DIMENSIONAMIENTO ÓPTIMO

En esta sección se presentan las principales ecuaciones que conforman el modelo matemático del programa de dimensionamiento óptimo de plantas cogeneradoras. Este modelo es en esencia un problema de optimización conformado por su función objetivo y su conjunto de restricciones. En los siguientes numerales se describen las ecuaciones y factores que conforman la función objetivo y las restricciones.

1.1 Función Objetivo

La función objetivo es representada como sigue:

$$\text{Costo} = Cc + Cp$$

donde,

Cc: Costo de capital. Este incluye el costo de los equipos, su importación, impuestos, instalación, puesta en marcha. Este valor se trabaja en pagos constantes horarios.

Cp: Costo de producción. Este incluye los costos de los combustibles comprados, la electricidad comprada y vendida, la operación y el mantenimiento hora a hora.

1.1.1 Costo de capital

El costo de capital se representa por medio de la siguiente ecuación:

$$Cc = CRFh * (1 - VSALV) * (Nu * CostoW + Nu * CostoWhrsg)$$

donde,

CRFh: Este factor convierte un valor presente en pagos constantes equivalentes durante una serie de periodos horarios y durante el tiempo de vida del proyecto.

VSALV: Este factor es el valor de salvamento como fracción del valor presente actual.

Nu: Número de unidades de generación.

CostoW: Costo de la unidad. Esta es una función del tamaño de la unidad, corregido por condiciones ambientales, y de los costos de importación como porcentaje del costo del equipo.

CostoWhrgs: Costo de la caldera de recuperación. Es función del tamaño de la caldera y de los costos de importación como porcentaje del costo del equipo.

A continuación se explican cada uno de los términos de la ecuación anterior.

1.1.1.1 CRFh

El CRF, Capital Recovery Factor, permite encontrar el valor equivalente de una anualidad futura equivalente dado un valor presente. El CRFh es equivalente al CRF en su formulación, no obstante su base es horaria.

$$CRFh = \frac{ih}{1-(1+ih)^{-VUTIL}}$$

donde,

*i*h: interés efectivo horario.

VUTIL: Vida útil en horas.

1.1.1.2 CostoW

El costo depende del tipo de planta que se esté tratando, esto es, de acuerdo con los precios de mercado y el tipo de equipo. Estos costos son los de la planta instalada en sitio.

Para máquinas diesel tenemos:

$$CostoWds = 1,15 * k * 1366,87 * W^{0,9138}$$

Para máquinas turbogas:

$$CostoWgt = 1,15 * k * 3965,11 * W^{0,7521}$$

Para máquinas turbovapor:

$$CostoWtv = k * 1,5 * \left(115 + 1700 * \left(4,4 + \frac{W}{1000}\right)^{-0,67}\right) * W$$

En cuanto a calderas de recuperación se tiene:

$$CostoWhrsg = k * 463,84 * W^{0,793}$$

donde,

CostoW: Es el costo en miles de pesos de 1995.

W: Capacidad o tamaño de la máquina en kW o en kBtu/h.
k: Es la relación del costo de las plantas FOB País de origen, CIF industria colombiana.

En el caso de ciclos combinados se usan los costos de las turbinas de gas, turbinas de vapor y calderas de recuperación conjuntamente. Cabe aclarar que de acuerdo con los avances tecnológicos, los costos pueden variar en términos reales.

1.1.2 Costo de producción

El costo de producción se representa por medio de la siguiente ecuación:

$$Cp = \varphi Ec * Ec - \varphi Ev * Ev + \varphi F1 * F1 + \varphi F2 * F2$$

donde,

φEc: Tarifa de compra de energía
Ec: Energía eléctrica comprada
φEv: Tarifa de venta de energía
Ev: Energía eléctrica vendida
φF1: Tarifa de compra de combustible F1
F1: Combustible F1 consumido
φF2: Tarifa de compra de combustible F2
F2: Combustible F2 consumido

1.2 Restricciones

Estas completan el planteamiento del problema. Se dividen en cuatro grupos principales: a) restricciones técnicas, b) restricciones operativas, c) restricciones físicas y/o comerciales, d)restricciones de validez.

1.2.1 Restricciones técnicas

Las restricciones técnicas se refieren a aquellas asociadas con las características de desempeño de los equipos, lo mismo que la configuración propia de la tecnología que se evalúa.

1.2.1.1 Consumo de combustible

Para ciclos de cogeneración basados en turbinas de gas se tiene:

$$\text{ConsumoF} = \text{HR}(\text{T},\%\text{carga},\text{W}) * \text{E}$$
$$= \left(0{,}9392 * (1{,}00134)^{\text{T}}\right) * \left(5{,}9369 * (100 * \text{E} / \text{W})^{-0{,}3919}\right) * \left(100 * 3{,}41214 / (7{,}8697 + 2{,}4333 * \ln(\text{W}))\right) * \text{E}$$

Para ciclos de cogeneración basados en máquinas diesel se tiene:

$$\text{ConsumoF} = \text{HR}(\%\text{carga},\text{W}) * \text{E}$$
$$= (3412{,}14 / 0{,}42) * \left(2{,}537 * (100 * \text{E} / \text{W})^{-0{,}2064}\right) * \text{E}$$

donde,

ConsumoF: Es el consumo en kBtu de combustible en una hora.
HR: HeatRate o consumo térmico específico en Btu/kWh
T: Temperatura en grados Fahrenheit.
E: Energía en kWh.
W: Capacidad de la planta en kW.
100*E/W: Porcentaje de carga en una hora.

Para ciclos basados en turbinas de vapor se tiene:

$$\text{ConsumoF} = \frac{1224{,}63 * \left(0{,}976 * (\text{E} / \text{W})^{-0{,}25}\right)}{0{,}92 * 1000 * \text{X}} * \text{E}$$

$$\text{X} = \sum_i \frac{\left(\dot{m}_i / \dot{m}_t\right)}{\text{ASR}_i},$$

$$\text{ASR}_i = 14{,}2855 + 9{,}916 * \text{P}_i / 100$$

donde,

E: Energía en kWh.
W: Capacidad de la planta en kW.
E/W: relación de carga en una hora en por unidad.
X: Son las extracciones
$\dot{m}_i$: Flujo de mas en lb/h de la extracción i-ésima
$\dot{m}_t$: Sumatoria de todos los flujo que salen de la turbina.
ASR_i: *Actual Steam Rate*, tasa de vapor real en lb/kWh, de la i-ésima extracción.
P_i: Presión en Psig de la i-ésima extracción.

1.2.1.2 Calor disponible

En turbinas de gas y motores diesel se tiene una cantidad de calor disponible para ser aprovechado, proporcional a la diferencia entre la energía ingresada por el combustible y la energía efectivamente generada.
El calor de escape en turbinas de gas y motores diesel es:

$$Q = q * (F - 3{,}41214 * E)$$

donde,

Q: Es el calor disponible en kBtu/h.
q: Es una constante de proporcionalidad que indica la fracción de calor realmente recuperable. En turbinas de gas se estima q=0,97. En motores diesel se tiene q=0,658.
F: Es la cantidad de combustible consumido en kBtu/h.
E: Es la energía eléctrica generada en kWh.

1.2.1.3 Calor recuperado a la salida de la caldera de recuperación

No todo el calor disponible de las turbinas de gas y de los motores diesel puede convertirse en calor útil. La siguiente ecuación refleja la porción de calor realmente útil que se encuentra a la salida de la caldera de recuperación.

$$\begin{aligned} Qhrsg &= Qhrsg(Whrsg, \%carga, Q) \\ &= Q * \left(23{,}33 + 13{,}4 * \ln(100 * Qhrsg / Whrsg) / 85{,}039\right) * \left(A - 1{,}37e - 6 * Whrsg\right) \end{aligned}$$

donde,

Q: Calor disponible de la turbina de gas o del motor diesel.
Whrsg: Capacidad de la caldera de recuperación en kBtu/h.
100*Qhrsg/Whrsg: Porcentaje de carga parcial.
A: Constante que refleja la eficiencia de la caldera de recuperación en función de la temperatura de salida de los gases o calor recuperable a la salida de la turbina de gas o del motor diesel. Para turbina de gas se tiene A=80,51, mientras que para motor diesel A=70,00.

1.2.1.4 Calor útil en turbinas de vapor

El caso de las turbinas de vapor se maneja con base en las libras/hora de vapor. Es de notar que esta relación ya hace parte de la ecuación de consumo de combustible y que este flujo es adecuado para las características de vapor demandadas en el proceso.

$$\dot{m}_{th} = \frac{0{,}92 * 1000 * F}{1224{,}63}$$

donde,

$\dot{m}_{th}$: Flujo de vapor útil en proceso

F: Combustible de entrada a la caldera que alimenta a la turbina de vapor.

1.2.1.5 Caldera auxiliar

El modelo considera además del vapor producido por la caldera de recuperación, vapor producido desde una caldera auxiliar. Esta caldera particular puede tomarse como una caldera ya existente.

$$Qba = 0{,}92 * F$$

donde,

Qba: Calor generado por la caldera auxiliar en kBtu/h.

F: Es la cantidad de combustible consumido en kBtu/h.

1.2.1.6 Caldera auxiliar para el caso de cogeneración con turbinas de vapor

Este caso también se maneja con libras/hora de vapor generadas envés de kBtu/h. Cabe anotar que se asume que la caldera auxiliar es de características similares a la caldera que alimenta la turbina de vapor. En este caso se tiene:

$$\dot{m}_{ba} = \frac{0{,}92 * 1000 * F}{1224{,}63}$$

donde,

$\dot{m}_{ba}$: Flujo de vapor útil en proceso

F: Combustible de entrada a la caldera auxiliar.

1.2.2 Restricciones operativas

Dentro de estas restricciones se encuentran las que asocian el desempeño de las plantas cogeneradoras y las demandas de la industria, es decir los balances de energía, y por otro lado los límites operativos propios de las máquinas.

1.2.2.1 Balances de energía

El balance de energía eléctrica se expresa como sigue:

$$Nu * Eg + Ec - Ev - CE = 0$$

donde,
Nu: Número de unidades generadoras.
Eg: Energía generada.
Ec: Energía comprada.
Ev: Energía vendida.
CE: Energía demandada en una hora.
El balance de energía térmica para plantas diesel y turbo gas se expresa así:

$$Qhrsg + Qba - CT = 0$$

donde,
Qhrsg: Es el calor a la salida de la caldera de recuperación en kBtu/h.
Qba: Es el calor a la salida de un caldera auxiliar en kBtu/h.
CT: Es la carga térmica en unidades consistentes.
En cuanto a plantas de vapor se tiene:

$$\dot{m}_{th} + \dot{m}_{ba} - CT \geq 0$$

donde,
$\dot{m}_{th}$: Es el flujo de vapor a la salida de la turbina de vapor para proceso, en lb/h
$\dot{m}_{ba}$: Es el flujo de vapor a la salida de la caldera auxiliar para proceso, en lb/h
CT: Es la carga térmica en unidades consistentes.

1.2.2.2 Límites operativos

A continuación se listan los límites establecidos en este problema de optimización.

$$0{,}35 \le E / W \le 1$$
$$0 \le Qhrsg / Whrsg \le 1$$
$$0 \le Qba / Wba \le 1$$
$$Whrsg \le W * (HR - 3412{,}14) / 1000$$

donde,

E:	Energía generada en kWh.
W:	Tamaño de la planta cogeneradora en kW.
Qhrsg:	Calor de salida de la caldera de recuperación en kBtu.
Whrsg:	Capacidad de la caldera de recuperación en kBtu/h.
Qba:	Calor de salida de la caldera auxiliar en kBtu.
Wba:	Capacidad de la caldera auxiliar en kBtu/h.
HR:	HeatRate o consumo térmico específico en kBtu/kWh

1.2.2.3 Restricciones físicas y/o comerciales

Las siguientes restricciones tienen que ver con límites físicos en la capacidad de suministro para la industria particular, al igual que a límites comerciales como cupos de compra de combustibles.

$$Ec \le Ecmax.$$
$$Ev \le Evmax.$$
$$F \le Fmax.$$

1.2.2.4 Restricciones de validez

Estas son las más evidentes de todas. Dictan los rangos dentro de los cuales son validas las ecuaciones de desempeño.

$$E \ge 0$$
$$F \ge 0$$
$$W \ge 0$$
$$Q \ge 0$$
$$50kW \le W \le 220000kW$$
$$1000kBtu / h \le Whrsg \le 8000000000kBtu / h$$
$$1000kBtu / h \le Wba \le 8000000000kBtu / h$$

1.3 Ciclo de Optimización

El problema de optimización planteado en las ecuaciones anteriores sirve, en una primera optimización, para establecer el conjunto de máquinas factibles durante el periodo de evaluación del proyecto, y posteriormente, en una segunda optimización, para evaluar la operación de cada una de estas máquinas a lo largo de su vida útil, y bajo las distintas condiciones dictadas por las demandas, tarifas y costos año a año.

Esta segunda optimización es la que finalmente permite hallar la mejor máquina dentro del conjunto de máquinas factibles.

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ANEXOS

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Optimal Sizing of a Gas Turbine Cogeneration Plant in Consideration of Its Operational Strategy

An optimal planning method is proposed for the fundamental design of cogeneration plants. Equipment capacities and utility maximum demands are determined so as to minimize the annual total cost in consideration of the plants' annual operational strategies for the variations of both electricity and thermal energy demands. These sizing and operational planning problems are formulated as a nonlinear programming problem and a mixed-integer linear programming problem, respectively. They are solved efficiently in consideration of their hierarchical relationship by a penalty method. A numerical example about a gas turbine plant is given to ascertain the validity and effectiveness of the proposed method.

Introduction

As dispersed and small-scale energy systems, cogeneration plants have been introduced increasingly for commercial and public purposes in Japan. In order to utilize their high economical and energy-saving potentials, planners are urged to propose good plans for the fundamental design of cogeneration plants. For example, it is considered an important subject to determine rationally equipment capacities and utility maximum demands, especially capacities of prime movers and a maximum demand of purchased electricity. This is because if the capacities of prime movers are underestimated, the effect of introducing cogeneration plants becomes relatively small, and contrarily if they are overestimated, the availability of prime movers decreases. However, this subject is very difficult to solve since it is necessary to take account of the plants' annual operational strategies for the variations of both electricity and thermal energy demands.

At present, a trial and error method is used conventionally to determine plants' sizes; i.e., economical and energy-saving properties are evaluated only for several alternatives regarding equipment capacities and utility maximum demands, among which the best alternative is selected. Additionally, the thermal- or electric-following strategies are adopted conventionally as operational strategies for prime movers. These conventional approaches have the disadvantage that the high economical and energy saving potentials cannot necessarily be utilized. Therefore, it is necessary to develop a rational method of determining plants' sizes and operational strategies.

From the above-stated viewpoint, the authors first proposed an optimal planning method of determining the operational strategies of cogeneration plants, and ascertained its effectiveness by carrying out case studies on several types of gas turbine plants (Ito et al., 1987, 1988, 1990a, 1990b; Ito and Yokoyama, 1989). However, it appears that there is no rational and practical method of determining the plants' sizes. For example, several authors proposed optimization approaches to the plant sizing by taking account of only a few energy demand patterns (Papoulias and Grossmann, 1983; Johnson et al., 1985; Horii et al., 1987). However, these approaches cannot apply directly to the sizing of cogeneration plants for commercial and public purposes, because the energy demands fluctuate seasonally and hourly, and it is necessary to consider hundreds of energy demand patterns.

The purpose of the present paper is to develop an optimal planning method of determining the sizes of cogeneration plants in consideration of their annual operational strategies. First, the main factors to be taken into account at the fundamental design stage are categorized, and the concept of the optimal planning method is described. Secondly, the optimization problem is formulated concretely for a simple cycle gas turbine cogeneration plant. Lastly, a numerical example is given about a plant used for district heating and cooling.

Optimal Planning Method

Main Factors in Fundamental Design. Economic or energy-saving properties are often used as evaluation criteria in the fundamental design of cogeneration plants, and they are influenced by many factors. Figure 1 shows these main factors categorized into (*a*) plant, (*b*) input energy, (*c*) output energy, (*d*) energy management, and (*e*) environment. These factors should be taken into account comprehensively. In this paper, the factors with the symbol ● or ◆ are regarded as design

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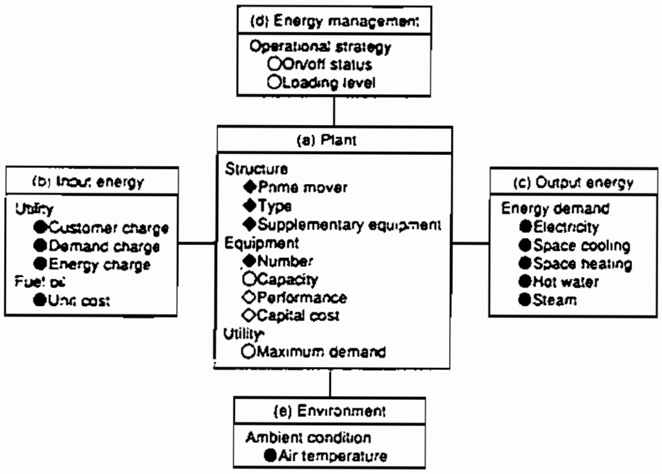

Fig. 1 Main factors to be taken into account in the fundamental design of cogeneration plants

specifications, those with ○ as design items to be determined in the design process, and those with ◇ as design characteristics to be determined together with the design items. Namely, equipment capacities and utility maximum demands are determined together with the plant's operational strategy for a given plant structure, utility tariff, energy demand, and ambient condition. Though the factors with ◆ are treated as design specifications here, they may also be regarded as design items. In this case, there exist many configurations regarding the plant structure. The method to be proposed is used to determine the size for each configuration and to select the best one.

Basic Concept of Optimal Planning Method. In this paper, the annual total cost is minimized from the viewpoint of long-term economics. It is evaluated as the sum of the annual capital cost and annual running cost on the basis of the annualized costs method (Witte et al., 1988). The annual capital cost of each piece of equipment is considered a function of its capacity. The annual running cost of each utility is the sum of the customer/demand charges and energy charge. The demand charge is considered a function of the utility maximum demand, and the energy charge is calculated from the plant's operational strategy.

As constraints, it is necessary to consider performance characteristics of each piece of equipment and energy balance relationships of each energy flow for average hourly energy demands estimated on several representative days in one year. In addition to the average energy demands, peak energy demands in summer and winter must be considered for equipment to supply energy during the peak periods. Here, since performance characteristics of equipment change with the capacities, they should be considered as functions of the capacities.

Design variables are composed of equipment capacities and utility maximum demands in the sizing problem, and the variables expressing the operational strategy in the operational planning problem. Though the capacity of each piece of equipment is selected from a set of discrete values in the practical design, it is regarded as a continuous variable in this paper. The operational strategy is expressed by the binary and continuous variables, which correspond to the on/off status of operation and energy flow rates, respectively (e.g., Ito et al., 1990a).

Hierarchical Optimization by Penalty Method. As stated in the introduction, the above planning problem is a large-scale and complex one since it includes the annual operational problem. It takes much computation time to optimize all the design variables simultaneously. In order to solve the problem efficiently, we propose a hierarchical optimization method: Equipment capacities and utility maximum demands are determined at the upper level; the operational strategy is assessed at the lower level; both levels are interconnected with each other by a penalty method.

This hierarchical optimization procedure is shown in Fig. 2. At the upper level, the optimal sizing problem is formulated as a nonlinear programming problem, in which optimal values of equipment capacities and utility maximum demands are searched so as to minimize the annual total cost. The constraints stated above are not taken into account explicitly, and only the constraints are considered that force virtual energy flows not to arise. At the lower level, for a plant with the

Nomenclature

Equipment symbols (subscripts)

- BA = gas-fired auxiliary boiler
- BW = waste heat recovery boiler
- EP = equipment for purchasing electricity
- GT = gas turbine generator
- PC = pump for supplying cold water
- RE = electric compression refrigerator
- RS = steam absorption refrigerator

Energy quantities

- E = electric power, MWh/h
- F = natural gas consumption (measured at standard conditions), m^3/h
- Q = heat flow rate, MWh/h

Other symbols

- C = initial capital cost of equipment, yen
- C_c = annual capital cost, yen/y
- C_d = annual customer and demand charges, yen/y
- C_e = annual energy charge, yen/y
- C_f = annual total cost, yen/y
- h = limit ratio of heat disposal to exhaust heat
- i = interest rate
- N = number of equipment installed
- p, q, r, s = performance characteristic values of equipment
- R = capital recovery factor
- v = ratio of salvage value to initial capital cost of equipment
- δ = binary variable expressing on/off status of operation
- η = size parameter of equipment
- κ = life of equipment, y
- λ = penalty cost, yen/MWh, yen/m^3
- φ = unit cost of utility energy charge, yen/MWh, yen/m^3
- Ψ = unit cost of utility customer charge, yen/month
- ψ = unit cost of utility demand charge, yen/(MW·month), yen/(m^3/h·month)
- $\overline{(\ )}$, $\underline{(\ )}$ = upper and lower bounds, respectively

Superscripts

- a = auxiliary machinery
- c = space cooling
- d = demand
- s = steam

Subscripts

- buy = purchased electricity
- disp = heat disposal
- gas = natural gas
- n = index for each piece of equipment
- PT = virtual energy flow

equipment capacities and utility maximum demands given at each searching step, the operational strategy is assessed and the annual energy charge is evaluated. In this paper, the operational strategy is assessed using an optimal operational planning method proposed by the authors (e.g., Ito et al., 1990a). Here, it may be done using such a conventional strategy as thermal- or electric-following strategies. In this procedure, virtual energy flows are added to the energy flows actually existing for any plant given at each searching step to satisfy the energy demands. The virtual energy flows play the role that they supply energy additionally only if the plant is too small to satisfy the energy demands. For that purpose, a penalty cost is imposed on the occurrence of virtual energy flows. At the upper level, given information about the occurrence of virtual energy flows, the design variables change their values automatically so as not to cause the virtual energy flows, or so as to satisfy the energy demands.

The introduction of this penalty method reduces drastically the number of design variables to be determined simultaneously, and consequently enables us to solve efficiently the sizing problem in consideration of the operational planning problem. The sequential linear programming method is adopted as a solving method of the nonlinear programming problem (Cheney and Goldstein, 1959). The optimal operational planning problem is formulated as a mixed-integer linear programming problem, and it is solved by the branch and bound method (Garfinkel and Nemhauser, 1972). The concrete algorithm adopted here is based on the Land-Doig method (Land and Doig, 1960).

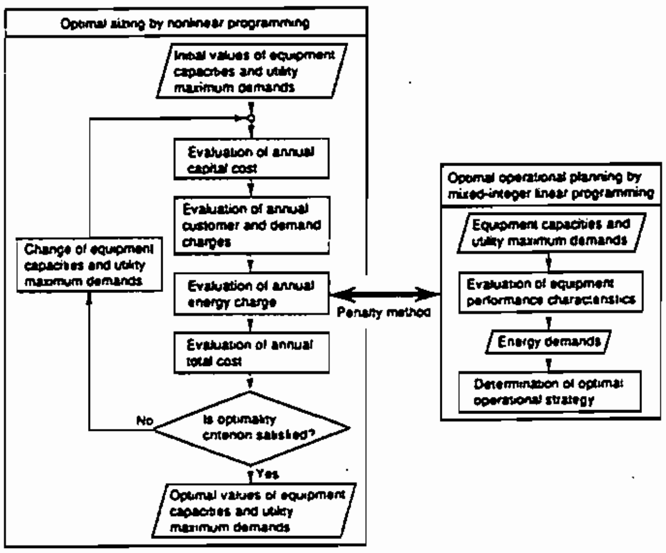

Fig. 2 A hierarchical algorithm of determining plants' sizes and operational strategies

Formulation for a Gas Turbine Plant

For a simple cycle gas turbine cogeneration plant, the optimal sizing problem is formulated together with the optimal operational planning problem.

Plant Structure. Figure 3 illustrates the structure of the plant investigated as an example. Equipment symbols are explained in the nomenclature. Although only one unit is illustrated for each kind of equipment, there may be several units. Solid lines, dot-dash lines, two dots-dash lines, dotted lines, and broken lines denote the flows of steam, electricity, cold water, exhaust gas, and natural gas, respectively. Bold arrows denote the virtual energy flows. An energy quantity is defined for each energy flow.

Electricity is supplied to users by the parallel running of gas turbine generators and purchasing electricity from an outside electric power company. Electricity is also used to drive electric compression refrigerators, pumps, and other auxiliary machinery in the plant. Exhaust heat generated by the gas turbines is recovered by waste heat recovery boilers, and it is reused for several kinds of thermal energy in a cascade way. The surplus exhaust heat is disposed of through the exhaust gas damper. The shortage of steam is supplemented by gas-fired auxiliary boilers. Cold water for space cooling is supplied by electric compression and steam absorption refrigerators. Steam is used for space heating and other miscellaneous purposes.

The capacities of each kind of equipment except the pumps are determined optimally together with the maximum demands of purchased electricity and natural gas. Here, it is assumed that the capacity of equipment for purchasing electricity is equal to the electricity maximum demand.

Optimal Sizing Problem. As stated above, the annual energy demands are satisfied by imposing constraints on the annual maximum values of virtual energy flows. That is, the constraints are expressed by

$$\left.\begin{array}{l} \max\limits_{t} E_{PT} \leq 0 \\ \max\limits_{t} Q^{c}_{PT} \leq 0 \\ \max\limits_{t} Q^{s}_{PT} \leq 0 \\ \max\limits_{t} F_{PT} \leq 0 \end{array}\right\}, \tag{1}$$

where $\max\limits_{t}$ means the annual maximum value.

The objective function is formulated in the following way. The annual capital cost of equipment C_c is expressed by

$$\begin{aligned} C_c = \{R(1-\upsilon)+i\upsilon\}\{&N_{GT}C_{GT}(\eta_{GT}) + N_{BW}C_{BW}(\eta_{BW}) \\ &+ N_{BA}C_{BA}(\eta_{BA}) + N_{RE}C_{RE}(\eta_{RE}) + N_{RS}C_{RS}(\eta_{RS}) \\ &+ C_{EP}(\eta_{EP})\}, \end{aligned} \tag{2}$$

where N is the number of pieces of equipment installed, C and η are respectively the initial capital cost and the size parameter

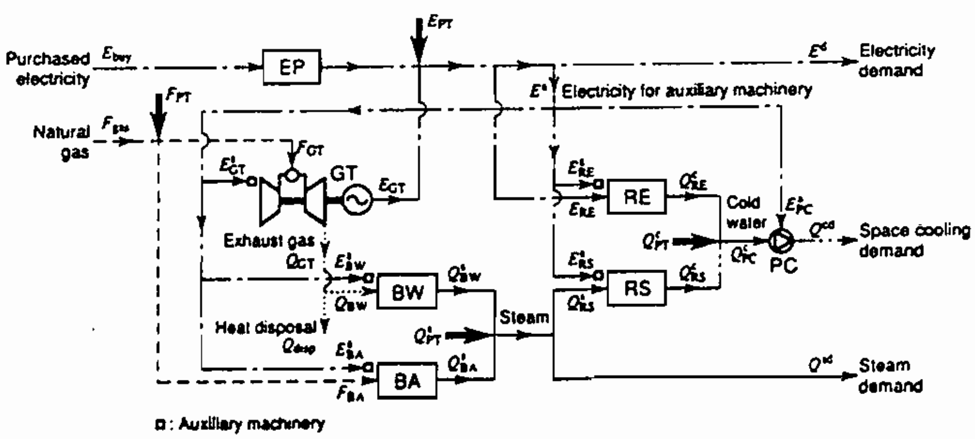

Fig. 3 Structure of a simple cycle gas turbine cogeneration plant

Table 1 Tariff of purchased electricity and natural gas

Utility	Unit cost		
	Customer charge	Demand charge	Energy charge
Purchased electricity	-	$\psi_{buy} = 1.74$ $\times 10^6$ yen/(MW·month)	$\varphi_{buy} = \begin{cases} 10.77 \text{ (July–Sept.)} \\ 9.79 \text{ (Other months)} \end{cases}$ $\times 10^3$ yen/(MWh)
Natural gas	$\Psi_{gas} = 36.0$ $\times 10^3$ yen/month	$\psi_{gas} = \begin{cases} 1.20 \text{ (Apr.–Oct.)} \\ 3.20 \text{ (Other months)} \end{cases}$ $\times 10^3$ yen/(m^3/h·month)	$\varphi_{gas} = 30.88$ yen/m^3

of each piece of equipment, υ is the ratio of salvage value to initial capital cost, and i is the interest rate. The capital recovery factor R is defined by

$$R = i(1+i)^\kappa / \{(1+i)^\kappa - 1\}, \quad (3)$$

where κ is the life of the equipment. In Eq. (2), it is assumed that the values of υ and κ are common to all kinds of equipment, and that several pieces of the same capacity are installed for each kind of equipment. The annual running cost is calculated as the sum of the customer/demand charges and the energy charge of purchased electricity and natural gas. Since there is the variety of tariff regarding these utilities, it is difficult to formulate the annual running cost generally. For the tariff shown in Table 1, for example, the customer/demand charges C_d and the energy charge C_e are respectively formulated as follows:

$$C_d = \sum_m \psi_{buy} \overline{E}_{buy} + \sum_m \Psi_{gas} + \sum_m \psi_{gas} \overline{F}_{gas} \quad (4)$$

and

$$C_e = \sum_t \varphi_{buy} E_{buy} + \sum_t \varphi_{gas} F_{gas}, \quad (5)$$

where $\overline{E}_{buy}$ and $\overline{F}_{gas}$ are, respectively, the maximum demands of purchased electricity and natural gas, and Ψ, ψ, and φ are, respectively, the unit costs of customer, demand, and energy charges. Symbols $\sum\limits_m$ and $\sum\limits_t$ denote the summations over annual total months and hours, respectively. As a result, the annual total cost C_t, the objective function to be minimized, is defined by

$$C_t = C_c + C_d + C_e. \quad (6)$$

The size parameters η_{GT}, η_{BW}, η_{BA}, η_{RE}, η_{RS}, η_{EP}, and the utility maximum demands $\overline{E}_{buy}$, $\overline{F}_{gas}$ are determined so as to minimize the objective function in Eq. (6), subject to the constraints in Eq. (1). In this procedure, the values of virtual energy flows in Eq. (1) and utility consumptions in Eq. (5) are calculated from the operational strategy. Any quantities may be adopted as size parameters, if they are representative of capacities. For example, maximum outputs are satisfactory as size parameters.

Optimal Operational Planning Problem. On the basis of the optimal operational planning method, the operational strategy is assessed so as to minimize the hourly energy charge for the given energy demands. The objective function in this problem is consistent with that in the optimal sizing problem. The following equations are fundamentally the same as those used in the optimal operational planning method (e.g., Ito et al., 1990a), and only a short comment is given here.

Performance characteristics are first formulated for each kind of equipment as follows:

Gas turbine generator

$$\left.\begin{array}{l} E_{GTn} = p_{GT}(\eta_{GT})F_{GTn} + q_{GT}(\eta_{GT})\delta_{GTn} \\ Q_{GTn} = r_{GT}(\eta_{GT})F_{GTn} + s_{GT}(\eta_{GT})\delta_{GTn} \\ E^e_{GTn} = p^e_{GT}(\eta_{GT})F_{GTn} + q^e_{GT}(\eta_{GT})\delta_{GTn} \\ \underline{F}_{GT}(\eta_{GT})\delta_{GTn} \le F_{GTn} \le \overline{F}_{GT}(\eta_{GT})\delta_{GTn} \\ \delta_{GTn} \in \{0, 1\} \end{array}\right\} (n = 1, \ldots, N_{GT}) \quad (7)$$

Waste heat recovery boiler

$$\left.\begin{array}{l} Q^s_{BWn} = p_{BW}(\eta_{BW})Q_{BWn} + q_{BW}(\eta_{BW})\delta_{BWn} \\ E^e_{BWn} = p^e_{BW}(\eta_{BW})Q_{BWn} + q^e_{BW}(\eta_{BW})\delta_{BWn} \\ \underline{Q}_{BW}(\eta_{BW})\delta_{BWn} \le Q_{BWn} \le \overline{Q}_{BW}(\eta_{BW})\delta_{BWn} \\ \delta_{BWn} \in \{0, 1\} \end{array}\right\} (n = 1, \ldots, N_{BW}) \quad (8)$$

Gas-fired auxiliary boiler

$$\left.\begin{array}{l} Q^s_{BAn} = p_{BA}(\eta_{BA})F_{BAn} + q_{BA}(\eta_{BA})\delta_{BAn} \\ E^e_{BAn} = p^e_{BA}(\eta_{BA})F_{BAn} + q^e_{BA}(\eta_{BA})\delta_{BAn} \\ \underline{F}_{BA}(\eta_{BA})\delta_{BAn} \le F_{BAn} \le \overline{F}_{BA}(\eta_{BA})\delta_{BAn} \\ \delta_{BAn} \in \{0, 1\} \end{array}\right\} (n = 1, \ldots, N_{BA}) \quad (9)$$

Electric compression refrigerator

$$\left.\begin{array}{l} Q^c_{REn} = p_{RE}(\eta_{RE})E_{REn} + q_{RE}(\eta_{RE})\delta_{REn} \\ E^e_{REn} = p^e_{RE}(\eta_{RE})E_{REn} + q^e_{RE}(\eta_{RE})\delta_{REn} \\ \underline{E}_{RE}(\eta_{RE})\delta_{REn} \le E_{REn} \le \overline{E}_{RE}(\eta_{RE})\delta_{REn} \\ \delta_{REn} \in \{0, 1\} \end{array}\right\} (n = 1, \ldots, N_{RE}) \quad (10)$$

Steam absorption refrigerator

$$\left.\begin{array}{l} Q^c_{RSn} = p_{RS}(\eta_{RS})Q^s_{RSn} + q_{RS}(\eta_{RS})\delta_{RSn} \\ E^e_{RSn} = p^e_{RS}(\eta_{RS})Q^s_{RSn} + q^e_{RS}(\eta_{RS})\delta_{RSn} \\ \underline{Q}^s_{RS}(\eta_{RS})\delta_{RSn} \le Q^s_{RSn} \le \overline{Q}^s_{RS}(\eta_{RS})\delta_{RSn} \\ \delta_{RSn} \in \{0, 1\} \end{array}\right\} (n = 1, \ldots, N_{RS}) \quad (11)$$

Pump for supplying cold water

$$\left.\begin{array}{l} E^e_{PCn} = p^e_{PC}Q^c_{PCn} + q^e_{PC}\delta_{PCn} \\ \underline{Q}^c_{PC}\delta_{PCn} \le Q^c_{PCn} \le \overline{Q}^c_{PC}\delta_{PCn} \\ \delta_{PCn} \in \{0, 1\} \end{array}\right\} (n = 1, \ldots, N_{PC}) \quad (12)$$

In these equations, for each kind of equipment the capacity of which is to be determined, performance characteristic values and lower/upper bounds of input energy are expressed as functions of the size parameter η.

From Fig. 3, energy balance and supply-demand relationships are formulated for each energy flow as follows:

Electricity

$$\left.\begin{array}{l} \sum\limits_{n=1}^{N_{GT}} E_{GTn} + E_{buy} + E_{PT} = \sum\limits_{n=1}^{N_{RE}} E_{REn} + E^e + E^d \\ E^e = \sum\limits_{n=1}^{N_{GT}} E^e_{GTn} + \sum\limits_{n=1}^{N_{BW}} E^e_{BWn} + \sum\limits_{n=1}^{N_{BA}} E^e_{BAn} \\ \quad + \sum\limits_{n=1}^{N_{RE}} E^e_{REn} + \sum\limits_{n=1}^{N_{RS}} E^e_{RSn} + \sum\limits_{n=1}^{N_{PC}} E^e_{PCn} \end{array}\right\} \quad (13)$$

Exhaust gas

$$Q_{GTn} = Q_{BWn} + Q_{dispn} \; (n = 1, \ldots, N_{GT}(=N_{BW})) \quad (14)$$

Table 2 Annual total and hourly maximum values of energy demands

	Annual total value GWh/y	Hourly maximum value MWh/h
Electricity	73.29	20.56
Space cooling	27.28	31.54
Steam	31.24	28.84

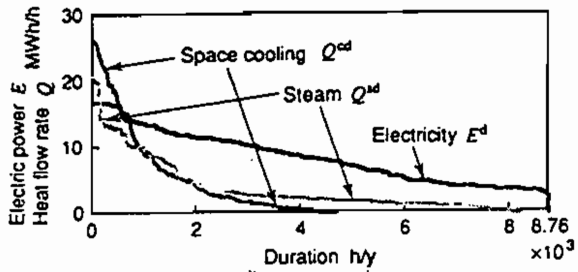

Fig. 4 Load duration curves for energy demands

Steam

$$\sum_{n=1}^{N_{BW}} Q^s_{BWn} + \sum_{n=1}^{N_{BA}} Q^s_{BAn} + Q^s_{PT} = \sum_{n=1}^{N_{RS}} Q^s_{RSn} + Q^{sd} \qquad (15)$$

Cold water

$$\sum_{n=1}^{N_{RE}} Q^c_{REn} + \sum_{n=1}^{N_{RS}} Q^c_{RSn} + Q^c_{PT} = \sum_{n=1}^{N_{PC}} Q^c_{PCn} = Q^{cd} \qquad (16)$$

Natural gas

$$F_{gas} + F_{PT} = \sum_{n=1}^{N_{GT}} F_{GTn} + \sum_{n=1}^{N_{BA}} F_{BAn} \qquad (17)$$

In addition, the following relationships must be satisfied between maximum demands and consumptions of utilities:

Electricity

$$E_{buy} \le \bar{E}_{buy} \qquad (18)$$

Natural gas

$$F_{gas} \le \bar{F}_{gas} \qquad (19)$$

The hourly energy charge, the objective function to be minimized, is expressed by

$$\varphi_{buy}E_{buy} + \varphi_{gas}F_{gas} + \lambda(E_{PT} + Q^c_{PT} + Q^s_{PT} + F_{PT}), \qquad (20)$$

where λ is the penalty cost, and it should be given a value much larger than the values of utility costs φ_{buy} and φ_{gas} in the numerical calculation.

The operational strategy is determined so as to minimize the objective function in Eq. (20), subject to the constraints in Eqs. (7)-(19). If a given plant satisfies the annual energy demands, all values of the penalty terms in Eq. (20) become zeros, and the constraint in the optimal sizing problem, Eq. (1), is satisfied. Otherwise, the penalty terms have a positive value, and consequently Eq. (1) is violated. However, in this optimization algorithm, the equipment capacities and utility maximum demands change their values automatically so as to satisfy Eq. (1).

Numerical Example

Input Data. The effectiveness of the proposed method is investigated through a numerical example about a simple cycle gas turbine cogeneration plant for district heating and cooling. In the district considered here, four office buildings and two hotels are to be constructed. The total floor area is about 384,000 m². For simplicity, it is assumed that the plant begins to supply energy to all buildings at the same time. Table 2 gives the annual total and hourly maximum values of electricity, space cooling, and steam demands. Figure 4 shows the load duration curves indicating the annual variations of hourly energy demands. Here, a weekday, a Saturday, and a holiday are considered representative days for each month; i.e., the operational strategy is investigated on 36 representative days throughout one year. Hourly energy demands are given as input data for each representative day. In addition to these average energy demands, peak energy demands in summer and winter are also given by multiplying the average energy demands on a weekday in August and January by 1.2, respectively. The functions expressing performance characteristic values and initial capital costs of equipment are determined from the actual data using the curve-fitting by the least squares method. The values shown in Table 1 are used for the tariff of purchased electricity and natural gas. In evaluating the annual capital cost, it is assumed that the ratio of salvage value to initial capital cost $v = 0$, the interest rate $i = 0.1$, and the life $\kappa = 15$ y.

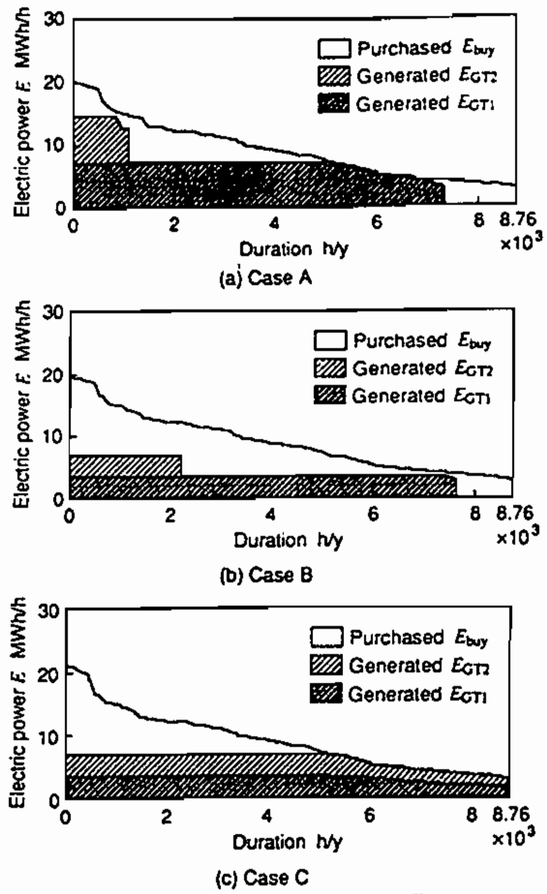

Fig. 5 Load duration curves for electricity supply

The economical merits of cogeneration plants can be divided into three parts; i.e., they are due to (*a*) introduction of cogeneration plants, (*b*) optimal operational planning, and (*c*) optimal sizing. Here, these merits are evaluated separately by carrying out the following four case studies:

- Case A: cogeneration plant with its size determined optimally by considering its optimal operational strategy
- Case B: cogeneration plant with its size given a priori by considering its optimal operational strategy
- Case C: cogeneration plant with its size given a priori by using the conventional electric-following strategy
- Case D: conventional plant composed of a gas-fired auxiliary boiler and a steam absorption refrigerator

Table 3 Values of equipment capacities and utility maximum demands

Case		A	B	C	D
Plant		Cogeneration			Conventional
Size		Optimal	Nonoptimal		
Operational strategy		Optimal	Optimal	Electric-following	–
Equipment capacity	Gas turbine generator $\bar{E}_{GT}$ MW	7.34	3.50	3.50	–
	Waste heat recovery boiler $\bar{Q}^{s}_{BW}$ MW	13.22	7.26	7.26	–
	Gas-fired auxiliary boiler $\bar{Q}^{s}_{BA}$ MW	2.41	14.34	14.34	28.84
	Electric compression refrigerator $\bar{Q}^{c}_{RE}$ MW	0.17	5.82	5.82	–
	Steam absorption refrigerator $\bar{Q}^{c}_{RS}$ MW	31.37	25.73	25.73	31.54
Utility maximum demand	Purchased electricity $\bar{E}_{buy}$ MW	9.54	18.27	18.27	26.10
	Natural gas $\bar{F}_{gas}$ $\times 10^3$ m³/h	4.70	3.76	3.76	2.77

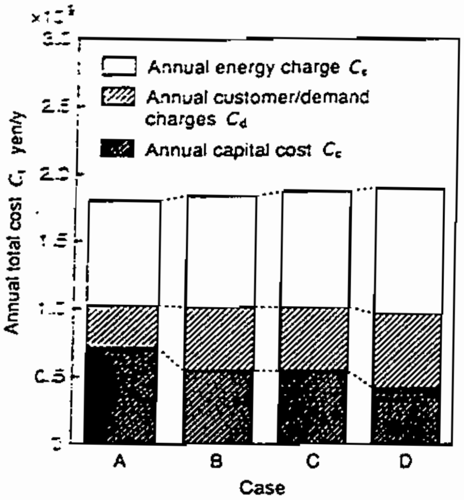

Fig. 6 Annual total cost and its items

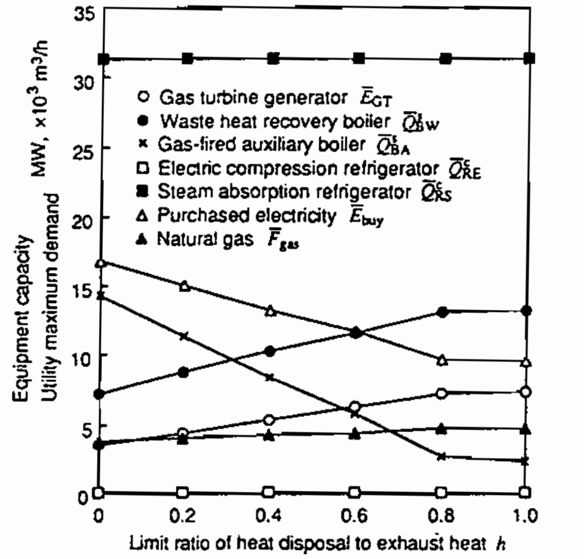

Fig. 7 Relationship between limit on heat disposal and optimal values of equipment capacities and utility maximum demands

It is assumed that two pairs of gas turbine generator/waste heat recovery boiler are installed in cases A to C, and that one unit is installed for other kind of equipment in all cases.

Results and Discussion. Table 3 shows the values of equipment capacities and utility maximum demands determined optimally for case A, and those values given a priori for cases B to D. For example, the capacity of gas turbine generators is discussed here. This value in case A is about twice as large as that in cases B and C. This result is understood from the annual operational strategy in the following way. Figure 5 shows the load duration curves for electricity supply, and (*a*) to (*c*) correspond to cases A to C, respectively. In case C, both gas turbine generators operate at base load status. In cases A and B, though the first gas turbine generator operates at base load status, the second one operates only at peak demand times. This is because by adopting the optimal operational strategy the second one stops so as not to dispose of a large amount of exhaust heat at times with low thermal energy demands. Additionally, the generating efficiency of gas turbine generators tends to increase with their capacity, and contrarily the thermal efficiency of exhaust heat recovery tends to decrease. As seen from Fig. 4, both space cooling and steam demands are relatively small as compared with the electricity demand in this district. Thus, in adopting the optimal operational strategy, gas turbine generators of larger capacity match this demand pattern.

Figure 6 shows the annual capital cost and its items for each case. In case D, though the annual capital cost is lower than those in other cases, the demand charge is higher because of its high maximum demand of purchased electricity, and the energy charge is also higher because of the conventional energy supply. In case C, in spite of a high capital cost, the introduction of the cogeneration plant leads to the reduction of both the demand and energy charges. In case B, the annual capital cost and the customer/demand charges are equal to those in case C, the energy charge decreases owing to the adoption of the optimal operational strategy. In case A, the annual total cost decreases further by optimizing both the plant's size and operational strategy. As a result, the reduction rate of annual total cost in cases A, B, and C to that in case D are 5.07, 3.43, and 1.50 percent, respectively. The differences in the annual total cost between cases C and D, B and C, A and B are due to the above-stated economical merits (*a*) to (*c*), respectively. This result shows that the economical merits can be obtained not only by introducing cogeneration plants but also by planning them optimally using the proposed method.

Although the result shown in Table 3 is determined optimally from the viewpoint of long-term economics, it is not satisfactory from the energy saving viewpoint. Namely, the capacity of gas turbine generators are so large that a large amount of exhaust heat must be disposed of. Hence, the optimization calculation is carried out again by adding the constraint that limits the heat disposal

$$Q_{dispn} \leq h\bar{Q}_{GTn} \quad (n = 1, \ldots, N_{GT}) \tag{21}$$

to the optimal operational planning problem. In Eq. (21), h is the limit ratio of heat disposal to exhaust heat generated from the gas turbine. Figure 7 shows the effect of the limit ratio h on the optimal values of equipment capacities and utility maximum demands. The constraint in Eq. (21) has little effect on the optimal values in the range of $0.8 \leq h \leq 1.0$. This is because a part of exhaust heat from the gas turbines is usually used for thermal energy supply. In the range of $h < 0.8$, however, the capacity of gas turbine generators decreases with the limit ratio h. At the point of $h = 0$, the capacity becomes about a half of that without limit on heat disposal. This result shows that the plant's size should be determined rationally by considering its operational strategy carefully.

Conclusions

An optimal planning method for the fundamental design of cogeneration plants has been proposed on the basis of the mathematical programming. The optimal sizing problem of determining equipment capacities and utility maximum demands has been described together with the optimal operational planning problem of determining the plants' annual operational strategies. An algorithm by a penalty method has been given to solve these problems efficiently in consideration of their hierarchical relationship. Through a numerical example about a simple cycle gas turbine cogeneration plant, it has turned out that equipment capacities and utility maximum demands can be determined rationally and easily by the proposed method. It has also proved that economical merits are not only due to introduction of cogeneration plants but also due to optimal sizing and operational planning by the method.

Although this paper has dealt with a numerical example only about a simple cycle gas turbine cogeneration plant, the proposed method applies not only to other types of gas turbine plants but also to plants with different prime movers. In addition to optimal planning by the method, it is important to accumulate, analyze, and arrange the data on performance characteristics of equipment and energy demands for various categories of buildings. It appears that these activities lead to significant findings on the fundamental design of cogeneration plants.

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Influence of Fuel Cost on the Operation of a Gas Turbine–Waste Heat Boiler Cogeneration Plant

The influence of fuel cost on the operation is investigated for a gas turbine-waste heat boiler cogeneration plant by an optimal operational planning method. A planning method is first presented by which the operational policy of each piece of constituent equipment is determined so as to minimize the operational cost. Then, a case study is performed for a cogeneration plant used for district heating and cooling. Through the study, it is made clear how the optimal operational policy and the economic or energy conservative properties are influenced by the costs of purchased electric power and natural gas. It is also shown that the optimal operational policy is superior in economy as compared with other conventional ones.

Introduction

Recently, cogeneration plants have been installed successfully in Japan into commercial and public buildings such as hotels, office buildings, hospitals, and so on. In addition, larger scale plants for district heating and cooling have also been installed into some areas with the redevelopment of cities and improvement in living standards.

The main advantage of introducing cogeneration plants is to be able to reduce operational costs and to save energy by efficient energy utilization. However, these objectives can be realized only if a good operational policy is adopted corresponding to energy demand. As cogeneration plants for industrial purposes have steady energy demand throughout the year, it is rather easy to determine the operational policy rationally. On the other hand, those for commercial and public purposes have an energy demand that fluctuates widely with time and seasons. In such a case, it is considered important at the planning stage to investigate in detail the operational policy corresponding to the fluctuating energy demand.

In the past, the thermal-following or electric-following policies have been adopted as conventional operational policies for cogeneration plants (California Energy Commission, 1982). According to these policies, prime movers are operated to follow the demand for thermal energy or electric power, respectively. However, they have the disadvantage that surpluses or shortages in supplies of electric power or thermal energy occur very often because of the lack of flexibility of operation. In addition, it appears to be irrational to adopt fixed operational policies throughout the year regardless of fuel cost and energy demand. From the economic viewpoint, it is rational to minimize the operational cost by changing the plant's operation flexibly corresponding to fuel cost and energy demand.

The objective of this paper is to propose such an optimal and flexible operational method for a gas turbine-waste heat boiler cogeneration plant. In order to investigate the operational policy of each piece of constituent equipment, it is necessary to take account of the discontinuity of performance characteristic due to the on/off status as well as the variation of efficiency due to the partial loading status (Akagi et al., 1986; Ducrocq, 1985; Nath and Holliday, 1985). As the fuel cost varies widely with its consumption, it is also important to investigate the influence of fuel cost on the operation. In this paper, a detailed investigation is made into the influence of fuel cost on the optimal operational policy and the economic or energy conservative properties through a case study on a cogeneration plant used for district heating and cooling. In addition, the economic and energy conservative properties based on the optimal operational policy are compared with those based on the thermal-following or electric-following ones.

Plant Structure

Figure 1 illustrates the structures of the plants investigated in this paper. The gas turbine-waste heat boiler cogeneration plant (plant A) shown in Fig. 1(*a*) is investigated as a representative type. A conventional energy supply plant (plant B) shown in Fig. 1(*b*) is also investigated to compare with plant A. The equipment symbols in Fig. 1 are explained in the nomenclature. Solid lines, dot-dash lines, two dots-dash

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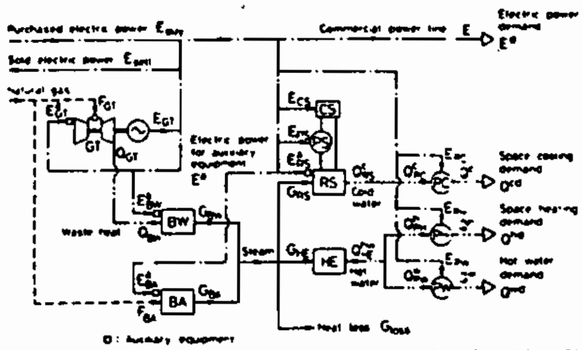

Fig. 1(a) Gas turbine-waste heat boiler cogeneration plant (plant A)

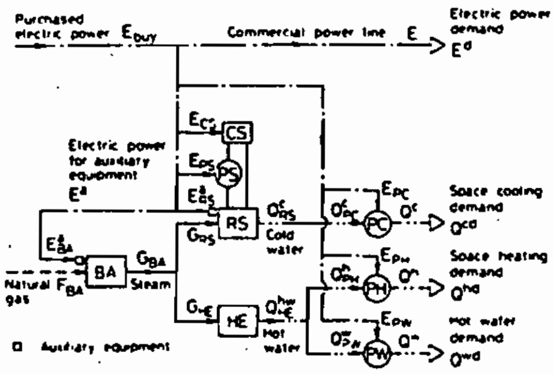

Fig. 1(b) Conventional energy supply plant (plant B)

Fig. 1 Plant structures

lines, and broken lines indicate the flows of steam, electric power, heat, and natural gas, respectively.

Plant A. Electric power is supplied to users by the parallel running of gas turbine generators and purchased electric power from an outside electric power company. The electric power is also used to drive pumps, cooling towers, and other auxiliary equipment in the plant. This plant has the capacity to sell the generated electric power to the outside electric power company. The waste heat generated by the gas turbines is recovered by waste heat boilers, and it is reused for several kinds of thermal energy in a cascade way. The surplus steam generated from wast heat boilers is disposed of through steam dumping, and it is included as heat loss in Fig. 1(*a*). The shortage of steam is supplemented by auxiliary boilers. Steam absorption refrigerators and heat exchangers are installed respectively to supply cold water for space cooling, and hot water for both space heating and hot water supply.

Plant B. Electric power is supplied to users only from purchased electric power, and thermal energy is supplied only by auxiliary boilers. The electric power is also used to drive auxiliary equipment in the plant.

Optimal Operational Planning Method

Formulation of Optimal Operational Planning Problem. As the electric power and thermal energy are supplied independently in plant B, the operational policy can be easily determined corresponding to their demands. On the other hand, as plant A is equipped with gas turbine generators supplying electric power and thermal energy simultaneously, there exist alternatives on the operational policy. In this case, it is difficult to determine the operational policy properly by the trial and error method. In this paper, an optimal operational planning method is proposed from the economic viewpoint, by which the operational policy is determined so as to minimize the operational cost.

A mathematical model is first presented to determine the operational policy. As operational policy, the on/off and par-

Nomenclature

Equipment symbols

- BA = auxiliary boiler
- BW = waste heat boiler
- CS = cooling tower for steam absorption refrigerator
- GT = gas turbine generator
- HE = heat exchanger
- PC = pump to supply cold water for space cooling
- PH = pump to supply hot water for space heating
- PS = pump to circulate cooling water for steam absorption refrigerator
- PW = pump to supply hot water
- RS = steam absorption refrigerator

Quantity of energy

- E = electric power
- F = natural gas consumption
- G = steam flow rate
- Q = heat flow rate

Other symbols

- C_r = annual operational cost
- f = monthly or annual load factor
- J = objective function
- l = annual rate of heat loss
- N = number of equipment installed
- p, q, r, s = parameters related to performance characteristic of each piece of equipment
- Q_p = annual consumption of primary energy
- α = unit cost ratio of purchased electric power to natural gas
- β = annual rate of energy conservation
- γ = reduction rate of annual operational cost
- δ = zero-one integer variable expressing on/off status of each piece of equipment
- Σ_t = summation over a month, a season, or a year
- $\varphi_{elec}, \varphi_{gas}$ = unit costs of purchased electric power and natural gas, respectively
- $\underline{(\ )}, \overline{(\ )}$ = lower and upper bounds of input or output energy of each piece of equipment, respectively

Superscripts

- a = electric power for auxiliary equipment
- c = space cooling
- d = demand
- h = space heating
- w = hot water supply

Subscripts

- A = plant A
- B = plant B
- buy = purchased electric power
- loss = heat loss
- n = identity number for each piece of equipment
- sell = sold electric power

tial loading status are investigated for each piece of constituent equipment. By relating the status respectively with zero-one integer and continuous variables, the optimal operational planning problem is formulated below for the gas turbine-waste heat boiler cogeneration plant shown in Fig. 1 (*a*).

Performance Characteristics of Equipment. The performance characteristics of each piece of equipment is formulated and considered as a constraint in this optimization problem.

(*a*) *Gas turbine generator* (*GT*). The performance characteristics of gas turbine generators can be expressed by relationships between electric power or flow rate of waste heat and natural gas consumption. The relationships can be approximated accurately enough by the following linear equations:

$$\left.\begin{array}{l} E_{GTn} = p_{GTn}F_{GTn} + q_{GTn}\delta_{GTn} \\ Q_{GTn} = r_{GTn}F_{GTn} + s_{GTn}\delta_{GTn} \\ \underline{F}_{GTn}\delta_{GTn} \le F_{GTn} \le \bar{F}_{GTn}\delta_{GTn} \\ \delta_{GTn} \in \{0, 1\} \end{array}\right\} (n = 1 \sim N_{GT}) \qquad (1)$$

where E, F, and Q denote electric power, natural gas consumption, and flow rate of waste heat, respectively. The coefficients p, q, r, and s denote parameters related to the performance characteristics. The symbols $(\_)$ and $(\bar{\ })$ denote lower and upper bounds of input or output energy, respectively. The zero-one integer variable δ expresses the on $(\delta = 1)$/off $(\delta = 0)$ status. The subscripts GT and n indicate that the amount is related to the nth of N_{GT} gas turbine generators. As the performance characteristics of gas turbine generators change with the intake air temperature, the coefficients and bounds in equation (1) are considered as functions of it. In addition, the electric power is needed to drive auxiliary equipment attached to gas turbine generators. It is also approximated by the following linear equation:

$$E^e_{GTn} = p^e_{GTn}F_{GTn} + q^e_{GTn}\delta_{GTn} \quad (n = 1 \sim N_{GT}) \qquad (2)$$

The performance characteristics of other pieces of equipment are similarly approximated by linear equations.

(*b*) *Waste heat boiler* (*BW*).

$$\left.\begin{array}{l} G_{BWn} = p_{BWn}Q_{BWn} + q_{BWn}\delta_{BWn} \\ E^e_{BWn} = p^e_{BWn}Q_{BWn} + q^e_{BWn}\delta_{BWn} \\ \underline{Q}_{BWn}\delta_{BWn} \le Q_{BWn} \le \bar{Q}_{BWn}\delta_{BWn} \\ \delta_{BWn} \in \{0, 1\} \end{array}\right\} (n = 1 \sim N_{BW}) \qquad (3)$$

(*c*) *Auxiliary boiler* (*BA*).

$$\left.\begin{array}{l} G_{BAn} = p_{BAn}F_{BAn} + q_{BAn}\delta_{BAn} \\ E^e_{BAn} = p^e_{BAn}F_{BAn} + q^e_{BAn}\delta_{BAn} \\ \underline{F}_{BAn}\delta_{BAn} \le F_{BAn} \le \bar{F}_{BAn}\delta_{BAn} \\ \delta_{BAn} \in \{0, 1\} \end{array}\right\} (n = 1 \sim N_{BA}) \qquad (4)$$

(*d*) *Steam absorption refrigerator* (*RS*), *cooling tower* (*CS*), *and pump to circulate cooling water* (*PS*).

$$\left.\begin{array}{l} Q^c_{RSn} = p_{RSn}G_{RSn} + q_{RSn}\delta_{RSn} \\ E^e_{RSn} = p^e_{RSn}G_{RSn} + q^e_{RSn}\delta_{RSn} \\ E_{CSn} = p_{CSn}G_{RSn} + q_{CSn}\delta_{RSn} \\ E_{PSn} = p_{PSn}G_{RSn} + q_{PSn}\delta_{RSn} \\ \underline{G}_{RSn}\delta_{RSn} \le G_{RSn} \le \bar{G}_{RSn}\delta_{RSn} \\ \delta_{RSn} \in \{0, 1\} \end{array}\right\} (n = 1 \sim N_{RS}) \qquad (5)$$

(*e*) *Heat exchanger* (*HE*).

$$\left.\begin{array}{l} Q^{hw}_{HEn} = p_{HEn}G_{HEn} \\ 0 \le G_{HEn} \le \bar{G}_{HEn} \end{array}\right\} (n = 1 \sim N_{HE}) \qquad (6)$$

(*f*) *Pump to supply cold water for space cooling* (*PC*)

$$\left.\begin{array}{l} E_{PCn} = p_{PCn}Q^c_{PCn} \\ 0 \le Q^c_{PCn} \le \bar{Q}^c_{PCn} \end{array}\right\} (n = 1 \sim N_{PC}) \qquad (7)$$

(*g*) *Pump to supply hot water for space heating* (*PH*).

$$\left.\begin{array}{l} E_{PHn} = p_{PHn}Q^h_{PHn} \\ 0 \le Q^h_{PHn} \le \bar{Q}^h_{PHn} \end{array}\right\} (n = 1 \sim N_{PH}) \qquad (8)$$

(*h*) *Pump to supply hot water* (*PW*).

$$\left.\begin{array}{l} E_{PWn} = p_{PWn}Q^w_{PWn} \\ 0 \le Q^w_{PWn} \le \bar{Q}^w_{PWn} \end{array}\right\} (n = 1 \sim N_{PW}) \qquad (9)$$

Energy Balance and Supply-Demand Relationships. From the plant structure shown in Fig. 1 (*a*), the following relationships are obtained for each energy flow. They are also considered as constraints.

(*a*) *Electric power.*

$$\left.\begin{array}{l} E_{buy} + E_{GT} = E_{sell} + E^e + E \\ E_{GT} = \sum_{n=1}^{N_{GT}} E_{GTn} \\ E^e = \sum_{n=1}^{N_{GT}} E^e_{GTn} + \sum_{n=1}^{N_{BW}} E^e_{BWn} + \sum_{n=1}^{N_{BA}} E^e_{BAn} \\ \quad + \sum_{n=1}^{N_{RS}} (E^e_{RSn} + E_{CSn} + E_{PSn}) + \sum_{n=1}^{N_{PC}} E_{PCn} \\ \quad + \sum_{n=1}^{N_{PH}} E_{PHn} + \sum_{n=1}^{N_{PW}} E_{PWn} \\ E = E^d \end{array}\right\} \qquad (10)$$

(*b*) *Steam.*

$$\sum_{n=1}^{N_{BW}} G_{BWn} + \sum_{n=1}^{N_{BA}} G_{BAn} = \sum_{n=1}^{N_{RS}} G_{RSn} + \sum_{n=1}^{N_{HE}} G_{HEn} + G_{loss} \qquad (11)$$

(*c*) *Cold water for space cooling.*

$$\left.\begin{aligned} &\sum_{n=1}^{N_{RS}} Q^{c}_{RSn} = \sum_{n=1}^{N_{PC}} Q^{c}_{PCn} = Q^{c} \\ &Q^{c} = Q^{cd} \end{aligned}\right\} \quad (12)$$

(*d*) *Hot water for space heating and hot water supply.*

$$\left.\begin{aligned} &\sum_{n=1}^{N_{HE}} Q^{hw}_{HEn} = Q^{h} + Q^{w} \\ &Q^{h} = \sum_{n=1}^{N_{PH}} Q^{h}_{PHn} \\ &Q^{w} = \sum_{n=1}^{N_{PW}} Q^{w}_{PWn} \\ &Q^{h} = Q^{hd} \\ &Q^{w} = Q^{wd} \end{aligned}\right\} \quad (13)$$

(*e*) *Waste heat.* Assuming that the plant is equipped with a paired gas turbine generator/waste heat boiler, the following relationship is taken into account:

$$Q_{GTn} = Q_{BWn} \quad (n = 1 - N_{GT}) \quad (14)$$

where $N_{GT} = N_{BW}$.

(*f*) *Natural gas.*

$$\left.\begin{aligned} &F_{GT} = \sum_{n=1}^{N_{GT}} F_{GTn} \\ &F_{BA} = \sum_{n=1}^{N_{BA}} F_{BAn} \end{aligned}\right\} \quad (15)$$

Objective Function. The objective function to be minimized is the hourly operational cost, defined by

$$J = \varphi_{elec}(E_{buy} - E_{sell}) + \varphi_{gas}(F_{GT} + F_{BA}) \quad (16)$$

where φ_{elec} and φ_{gas} denote the unit costs of purchased electric power and natural gas, respectively. Considering the case where the electric power company operates the cogeneration plant, the unit cost of sold electric power is assumed to be equal to that of purchased electric power.

The optimal operational planning problem has resulted in a mixed-integer linear programming one (Garfinkel and Nemhauser, 1972), in which the zero-one integer and continuous variables expressing the operational policy are determined so as to minimize J in equation (16) subject to equations (1) to (15).

Solving Method. The branch and bound method is adopted to solve the optimization problem defined above (Garfinkel and Nemhauser, 1972). This method is composed of the following three operations: (*a*) branching: the set of feasible solutions, or the feasible region, is divided into several subsets based on the values of integer variables; (*b*) solving linear programming problems: the suboptimal solution is derived for each subset using the simplex algorithm, in which calculations are carried out by simplex tableaus expressing the objective function and constraints (Dantzig, 1963); (*c*) bounding: by calculating a lower bound of the objective function for each subset, the subset is excluded from the feasible region if it becomes clear that there exists no optimal solution in this subset. Using this mehtod, the optimal solution can be derived more effectively than by the conventional enumeration method. The algorithm used in this paper is based on the Land-Doig method (Land and Doig, 1960; Kuester and Mize, 1973). A detailed explanation of the algorithm is omitted here.

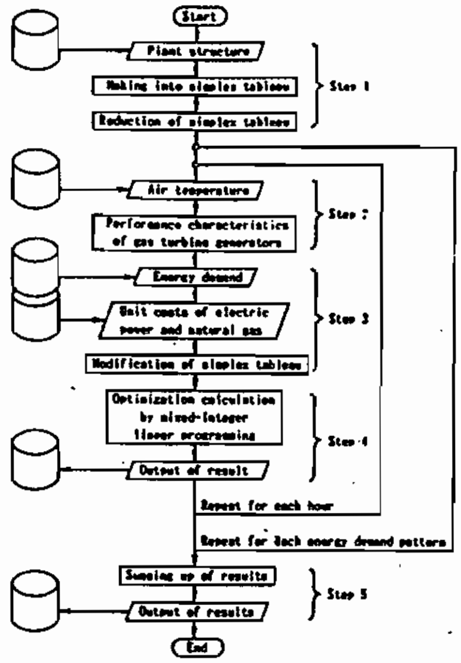

Fig. 2 Systematic procedure to determine the optimal operational policy

Systematic Procedure. Figure 2 shows a systematic procedure organized to determine the optimal operational policy throughout the year by using the above mentioned mathematical model. Each step in the procedure is described below:

Step 1: The input data on the plant structure are made into the simplex tableau required for mixed-integer linear programming. Here, the input data are expressed in the form of character expressions that enable easy and flexible modification of the data with a change of plant structure. Then, for the purpose of the efficient optimization calculation, the size of the simplex tableau is reduced by eliminating dependent variables automatically.

Step 2: The coefficients p_{GTn}, q_{GTn}, r_{GTn}, s_{GTn} and the bounds $\underline{F}_{GTn}$, $\overline{F}_{GTn}$ in equation (1) are determined according to the hourly input data on the air temperature.

Step 3: The simplex tableau is modified according to the hourly input data on energy demand and unit costs of purchased electric power or natural gas.

Step 4: The optimal operational policy is determined by the mixed-integer linear programming method, and the result is stored hourly.

Step 5: The hourly results are summed up for each month, each season, or the year, and the economic and energy conservative properties are estimated.

Steps 2 to 4 are repeated hourly for each energy demand pattern given on a representative day.

Case Study

Input Data. A case study is performed for a gas tur-

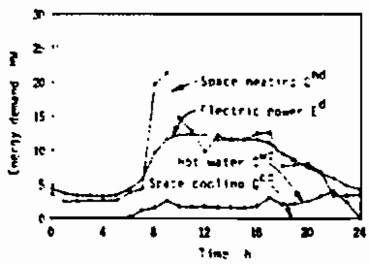

Fig. 3(a) Ordinary day in January

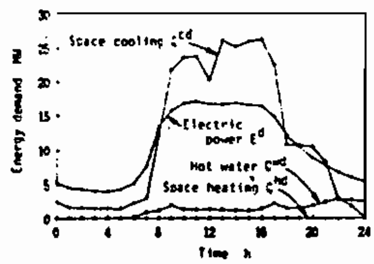

Fig. 3(b) Ordinary day in August

Fig. 3 Energy demand patterns

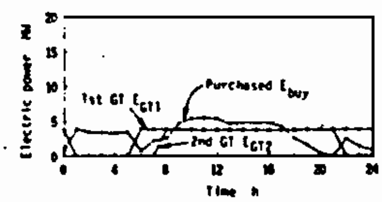

Fig. 4(a) Ordinary day in January

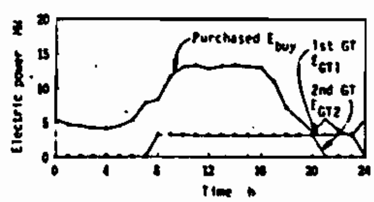

Fig. 4(b) Ordinary day in August

Fig. 4 The optimal operational patterns of gas turbine generators and purchased electric power

bine-waste heat boiler cogeneration plant used for district heating and cooling. The area supplied with energy consists of four office buildings and two hotels. Ordinary days, Saturdays, and holidays are considered as representative days for each month; i.e., the operational policy is investigated on 36 representative days throughout the year. Hourly energy demand is given as input data for each representative day. For example, Figs. 3(*a*) and 3(*b*) show the energy demand patterns estimated on the ordinary days in January and August, respectively. Two alternatives on the structure of plant A are investigated here; i.e., one is equipped with one pair of gas turbine generator/waste heat boilers (plant A1), and the other is equipped with two pairs (plant A2). Table 1 shows the number and maximum output of each kind of equipment in-

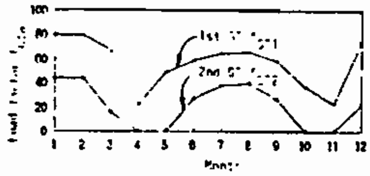

Fig. 5 Seasonal fluctuation of the monthly load factor of gas turbine generators

Table 1 Number and maximum output of each kind of constituent equipment

		Plant		
		A1	A2	B
Constituent equipment	Maximum output	Number	Number	Number
Gas turbine generator (GT)	$\bar{E}_{GTn}=3.45$ MW, $\bar{Q}_{GTn}=7.90$ MW (intake air temp. 17.5°C)	$N_{GT}=1$	$N_{GT}=2$	-
Waste heat boiler (BW)	$\bar{G}_{BWn}=2.97$ kg/s*	$N_{BW}=1$	$N_{BW}=2$	-
Auxiliary boiler (BA)	$\bar{G}_{BAn}=4.17$ kg/s*	$N_{BA}=3$	$N_{BA}=2$	$N_{BA}=3$
Steam absorption refrigerator (RS)	$\bar{Q}_{RSn}=5.83$ MW *	$N_{RS}=6$	$N_{RS}=6$	$N_{RS}=6$
Heat exchanger (HE)	$\bar{Q}_{HEn}=14.7$ MW	$N_{HE}=2$	$N_{HE}=2$	$N_{HE}=2$

* 0.690 MPa gauge saturated steam

stalled in plants A1, A2, and B. The details of input data on performance characteristics of equipment are omitted here.

The objective function in equation (16) indicates that the optimal operational policy can be changed by the unit costs of purchased electric power and natural gas. The unit costs of purchased electric power and natural gas vary widely with their demands. In this case study, the investigation is made into the influence of the unit cost ratio of purchased electric power to natural gas on the optimal operational policy and the economic or energy conservative properties. Defining the unit cost as the cost per unit consumption of primary energy, the unit cost ratio α is given by

$$\alpha = \frac{\varphi_{elec}[\text{Yen/MJ}] \times 0.35}{\varphi_{gas}[\text{Yen/kg}]/49.60[\text{MJ/kg}]} \quad (17)$$

As expressed in equation (17), the overall thermal efficiency of purchased electric power is assumed as 35 percent. The optimal operational policy and other properties are investigated parametrically with regard to the unit cost ratio α.

Results and Considerations.

Operational Pattern. The optimal operational policy of plant A2 has been first investigated in detail for the case of the unit cost ratio $\alpha = 0.952$. Figures 4(*a*) and 4(*b*) show the optimal operational patterns of gas turbine generators and purchased electric power on the ordinary days in January and August, respectively. Both gas turbine generators are operated at the high loading level in the daytime in January and August because of the high demand for thermal energy. Figure 5 shows the seasonal fluctuation of the monthly load factor of two gas turbine generators given by

$$f_{GTn} = \left(\sum_t E_{GTn}\right) \Big/ \left(\sum_t \bar{E}_{GTn}\right) \times 100 \quad (n = 1 \sim N_{GT}) \quad (18)$$

where Σ denotes summation over a month, a season, or a year. The monthly load factor f_{GTn} varies widely throughout the year for both gas turbine generators. The monthly load factor of the second gas turbine generator f_{GT2} is reduced to zero in spring and autumn. Thus, the investigation into the seasonal and time fluctuation of the load factor of prime movers proves to be a very important subject at the planning stage of cogeneration plants for commercial and public purposes.

Influence of Fuel Cost on Operational Policy. Next, an investigation has been made into the influence of the unit cost

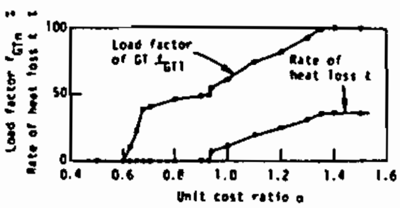

Fig. 6(a) Plant A1

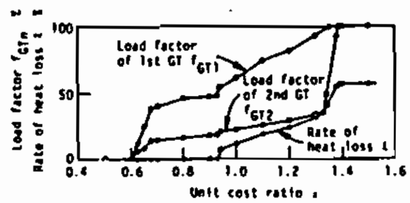

Fig. 6(b) Plant A2

Fig. 6 Relationships between the unit cost ratio and the annual load factor of gas turbine generators, or the annual rate of heat loss

ratio α in equation (17) on the optimal operational policy. Figure 6 shows the relationships between the unit cost ratio α and the annual load factor of gas turbine generators f_{GTn} in equation (18), or the annual rate of heat loss relative to the heat recovered by waste heat boilers l given by

$$l = \left(\sum_t G_{loss}\right) \Big/ \left(\sum_t \sum_{n=1}^{N_{BW}} G_{BWn}\right) \times 100 \qquad (19)$$

Figures 6(a) and 6(b) correspond to plants A1 and A2, respectively.

The results for plant A1 are described first. In the region of $\alpha \leq 0.6$, the annual load factor of the gas turbine generator f_{GT1} becomes zero. This indicates that the conventional operation is superior in economy as compared with the cogeneration operation because of the very low cost of purchased electric power. In the region of $0.6 < \alpha \leq 0.93$, f_{GT1} increases with α. Nevertheless, the annual rate of heat loss l is equal to zero. According to this annual total result, the optimal operational policy in this region resembles the thermal-following one. In the region of $0.94 \leq \alpha \leq 1.35$, f_{GT1} increases gradually with α in spite of the generation of a large amount of heat loss. In this region, several kinds of operational policies coexist with one another; i.e., the conventional or thermal-following policies are dominating during times of low demand for thermal energy, and the electric-following or sending back (i.e., selling the generated electric power to the outside electric power company) policies are dominating during times of high demand for thermal energy. Both the load factor f_{GT1} and the rate of heat loss l increase with α due to the gradual domination of the electric-following or sending back policies. In the region of $\alpha > 1.35$, f_{GT1} amounts to 100 percent.

Next, the results for plant A2 are described. In the region of $\alpha \leq 1.33$, the annual load factor of the first gas turbine generator f_{GT1} and the annual rate of heat loss l almost agree with those of plant A1, respectively. As the second gas turbine generator is operated according to the thermal-following policy so as not to generate the heat loss, its annual load factor f_{GT2} is very low. However, in the region of $1.33 < \alpha \leq 1.38$, f_{GT2} increases drastically with α due to the domination of the electric-following or sending back policies. In the region of $\alpha > 1.38$, f_{GT2} also amounts to 100 percent. Thus, with the increase in the unit cost ratio α, the optimal operational policy changes gradually by adopting partially the suitable policies from several kinds of operational policy.

Influence of Fuel Cost on Operational Cost. An investigation has been made into the influence of the unit cost ratio α on an economic property. Figure 7 shows the relationship between the unit cost ratio α and the reduction rate of annual operational cost γ defined by

$$\gamma = (C_{vB} - C_{vA})/C_{vB} \times 100 \qquad (20)$$

where C_{vA} and C_{vB} denote, respectively, the annual operational cost of plants A and B, given by

$$C_v = \sum_t J \qquad (21)$$

Figures 7(a) and 7(b) correspond to plants A1 and A2, respectively. As shown in equation (20), γ indicates how much the annual operational cost can be reduced by employing the cogeneration plant instead of the conventional energy supply plant. In Fig. 7, the optimal operational policy is compared with the thermal-following or electric-following ones. The value of γ increases with α. This is because the cogeneration operation becomes advantageous with the increase in α due to the low cost of natural gas. The thermal-following or electric-following policies approach the optimal one within a partial range of α. However, the optimal one is superior in economy for all values of the unit cost ratio α as compared with other ones.

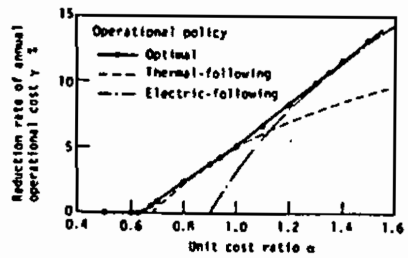

Fig. 7(a) Plant A1

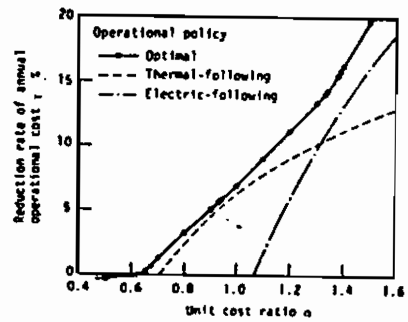

Fig. 7(b) Plant A2

Fig. 7 Relationship between the unit cost ratio and the reduction rate of annual operational cost

Influence of Fuel Cost on Energy Conservation. This paper defines the optimal operational policy as the one that minimizes the operational cost. In addition, as the energy conservative property is one of other important estimation indicators for cogeneration plants, it has also been investigated. Figure 8 shows the relationship between the unit cost ratio α and the annual rate of energy conservation β defined by

$$\beta = (Q_{pB} - Q_{pA})/Q_{pB} \times 100 \qquad (22)$$

where Q_{pA} and Q_{pB} denote, respectively, the annual consumption of primary energy of plants A and B, given by

$$Q_p = \sum_t \{E_{buy}/0.35 + (F_{GT} + F_{BA}) \times 49.60\} \qquad (23)$$

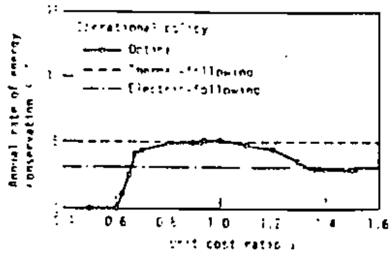

Fig. 8(a) Plant A1

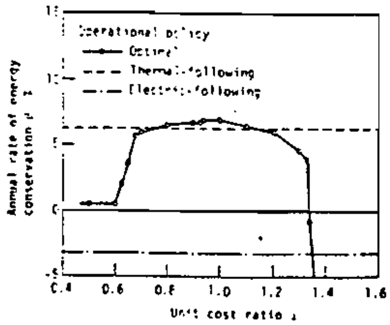

Fig. 8(b) Plant A2

Fig. 8 Relationship between the unit cost ratio and the annual rate of energy conservation

Figures 8(a) and 8(b) correspond to plants A1 and A2, respectively. The optimal operational policy is compared with the thermal-following or electric-following ones, which have a constant value of β due to the independence of the operational policy on α. According to the optimal operational policy, β has a maximum in the vicinity of $\alpha = 1$. In the region of $\alpha < 1$, β decreases because of the low load factor of gas turbine generators. On the other hand, in the region of $\alpha > 1$, β decreases because of the high rate of heat loss. This can be interpreted more exactly as follows. The value $\alpha = 1$ indicates that the unit costs of purchased electric power and natural gas are equal to each other. Hence, the objective function defined by equation (16) also indicates the minimization of the consumption of primary energy provided that no generated electric power is sold. Namely, β has a maximum at $\alpha = 1$ on the above assumption. In this case study, the value of β at $\alpha = 1$ reaches nearly its maximum because of the very little amount of sold electric power. The value of β on the basis of the thermal-following policy is close to the maximum. However, the value of β on the basis of the electric-following policy is far lower, and plant A2 has the negative value of β.

Conclusions

An optimal operational planning method has been proposed for a gas turbine-waste heat boiler cogeneration plant, and by this method an investigation has been made into the influence of fuel cost on the operation. A planning method has been first presented by which the operational policy is determined systematically throughout the year so as to minimize the operational cost. Then, a case study has been performed for a cogeneration plant used for district heating and cooling. The following main results have been obtained:

1 The optimal operational planning method enables easy and rational determination of the operational policy for a cogeneration plant with complex structure.

2 It has been made clear how the optimal operational policy and the economic or energy conservative properties are influenced by the unit cost ratio of purchased electric power to natural gas.

3 The optimal operational policy is superior in economy as compared with the thermal-following or electric-following ones.

Although this paper has dealt with a restricted case study, it appears that the obtained results apply qualitatively to other cases. In addition, the proposed method has the flexibility to apply to investigation into other cases. As a subsequent subject, it will be important to investigate the optimal capacity of cogeneration plants.

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